Influence of Risk Aversion on Optimal Irrigation Management Strategies

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Presentation Outline

- Introduction
  - Irrigation in Georgia
  - Understanding of why water use for irrigation (Who benefits? Is irrigation used to reduce risk? Or increase profits?)
  - What is the nature and property of irrigators' utility function (risk neutral or risk averse?)
- Methodology

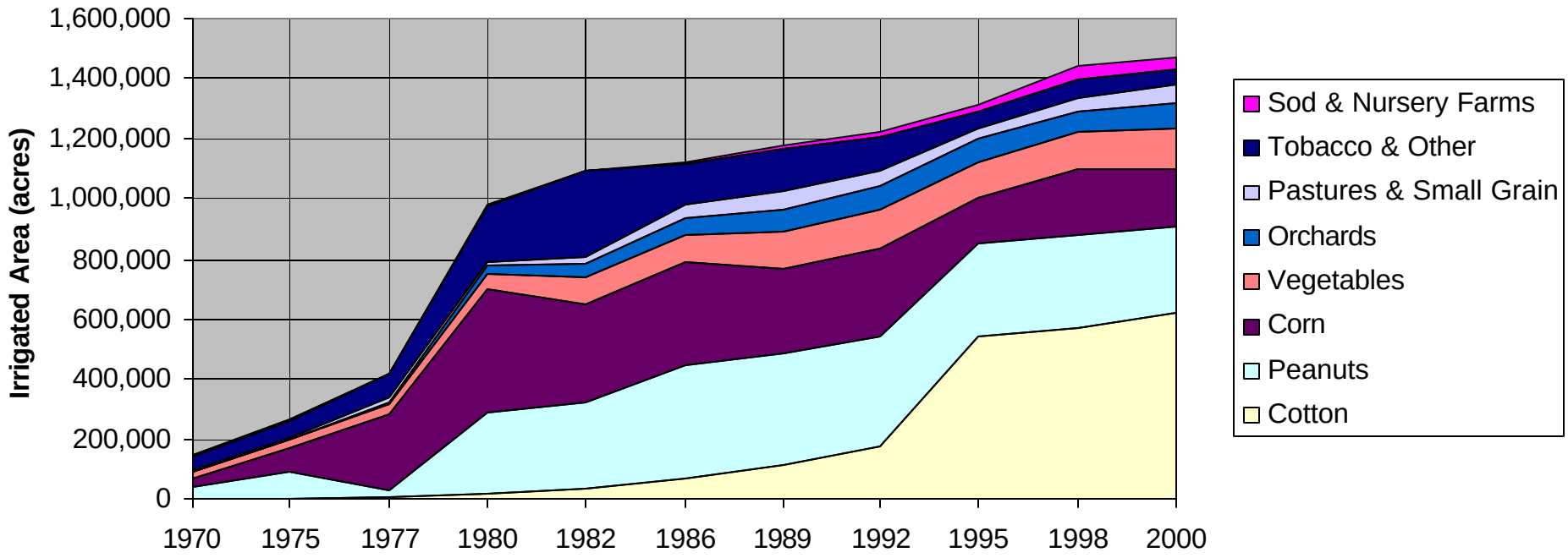
  - Mean-variance Utility Models

  - Crop Growth Simulation Model
- Results and Discussions

Why irrigation in a humid area

- Economic benefit to region (Makes land much more productive)
- Allows year-round production
- Multiple efficiencies (land & fertilizers)
- Risk aversion (Reduces / alleviates potential risk of crop failure due to climate change; Makes yield less volatile)

Irrigation Growth timeline



Trends in Irrigation

- Towards water application efficiency
- Upgrades in existing equipment (save money through energy savings)
- Precision timing and application
- Limitations:
  - **Limited attention to the behavior of agricultural producers under risk**
  - **Not clear about individual risk preference**

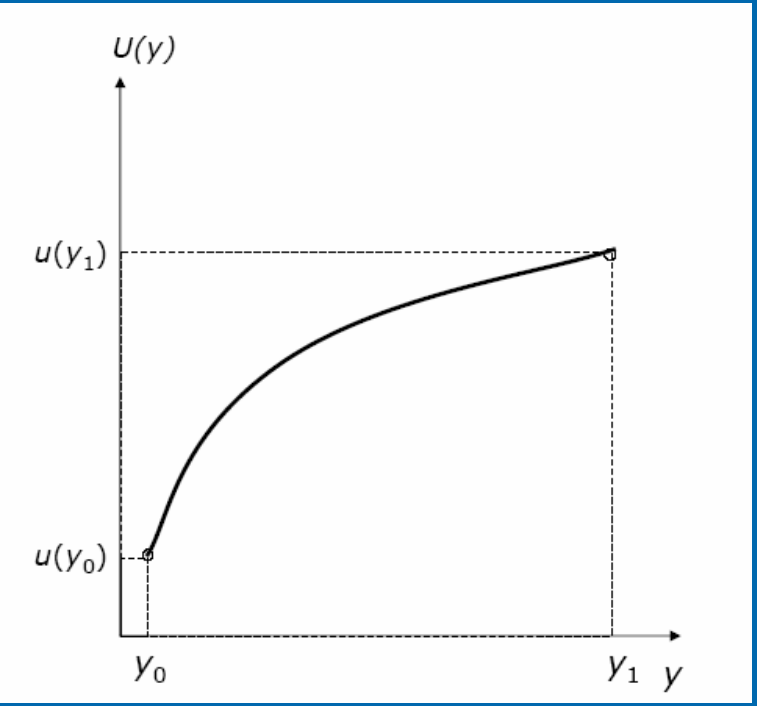
Objectives of this Presentation

- Investigate two utility functions: CARA and CRRA (with DARA)
- Optimize CARA and DARA wrt irrigation over range of risk aversion
- Examine potential impact of changes in peanut price support policy on water use

METHODOLOTY

Production Models in the Presence of Risk

- Maximize farmers' utility
  - Risk Averse: concave utility
    $u' > 0; u'' < 0$
  - Risk Neutral: linear utility
- Risk aversion is shown by the degree of concavity of the utility function
- We define the Arrow-Pratt measure of Absolute Risk Aversion (ARA) as a local measure of the degree that an agent dislikes risk.



$$R_a(y) = -\frac{u''}{u'}$$

Expected Utility (EU)

- Maximize the expected value of a von Neumann-Morgenstern utility function of profit

$$EU(p) = EU(\mathrm{P*Y\ (d,\ i)\ -\ C(d,\ i)})$$

- E : expectation operator
- P : product price,
- Y : crop yield (kg/ha),
- C: total cost,
- d: irrigation threshold,
- i: the year of simulation.

Mean-variance (E-V) to express Expected Utility

- Suppose farm profit is a set of random variables generated by X, then any profit is equal in distribution to:

$$m_p + s_p X$$

- The expected utility

$$E[U(p)] = \int_a^b u(m + s\,x)\,dF(x) = V(m_p, s_p)$$

Constant Absolute Risk Aversion CARA

$$V(m,s) = m - \frac{rs^2}{2}$$

Where r is the coefficient of absolute risk aversion.
We can easily calculate

$$S(m,s) = -\frac{V_s(m,s)}{V_m(m,s)} = rs$$

$S_m = 0$, meaning constant absolute aversion.

$$S_t = \frac{\partial S(tm,ts)}{\partial t} = \frac{\partial rts}{\partial t} = rs > 0$$

,meaning increasing relative risk aversion.

Constant Relative Risk Aversion CRRA

$$V(m,s) = -(m^2 - l\,s^2)^{-1}$$

Where ? is the coefficient of relative risk aversion.

$$S = -\frac{V_s}{V_m} = \frac{(m^2 - l\,s^2)^{-2}\,2\,l\,s}{(m^2 - l\,s^2)^{-2}\,2m} = \frac{l\,s}{m}$$

$$S_m = -\frac{l\,s}{m^2} < 0$$

, meaning decreasing absolute aversion.

$$S_t = \frac{\partial S(tm, ts)}{\partial t} = 0$$

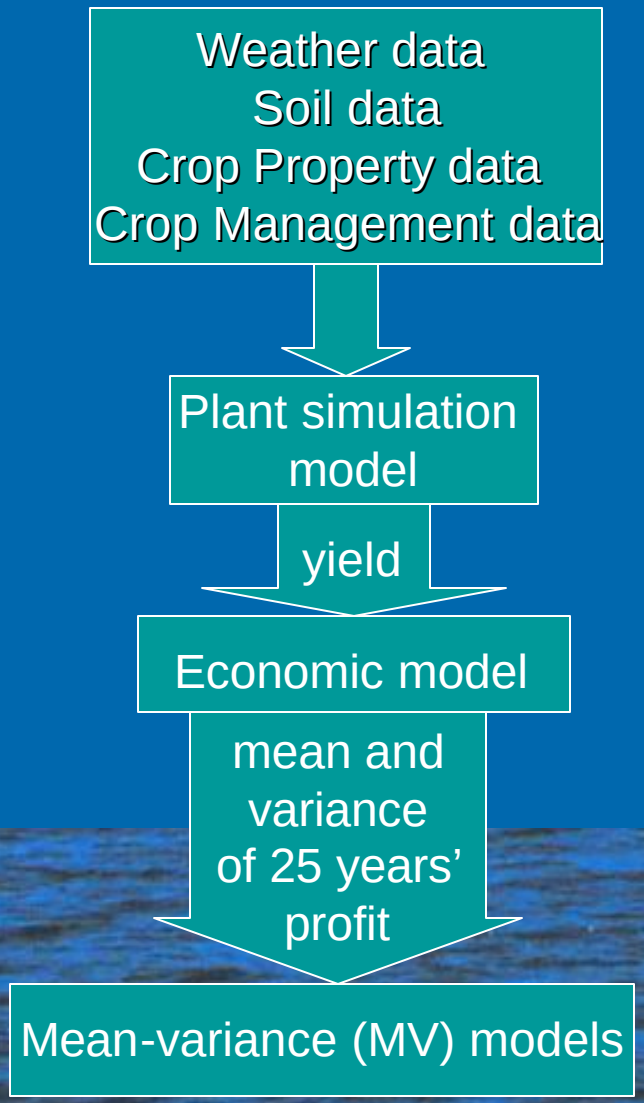
, meaning constant relative risk aversion.

Optimal Irrigation Level

- Use Decision Support System for Agro-Technology Transfer (DSSAT) to simulate yields over 25 years for different irrigation strategies.

- Adjust risk aversion coefficients from zero in small steps to find their critical points:

  - where a very small ?'s in risk aversion coefficient would lead to ?'s in irrigation decision.



Input Data for DSSAT

- Weather Data: daily solar radiation, daily maximum and minimum temperatures, and precipitation.
- Soil: Millhopper fine sand in Florida; Wagram Sand, Tifton Loam Sand, and Norfolk Loam Sand in Georgia
- Cultivar : Flounder in FL; Georgia Green in GA.
- Irrigation Strategies: 15 levels of Irrigation threshold.
- Each strategy is simulated over the series of 25 growing years (from 1976-2000).

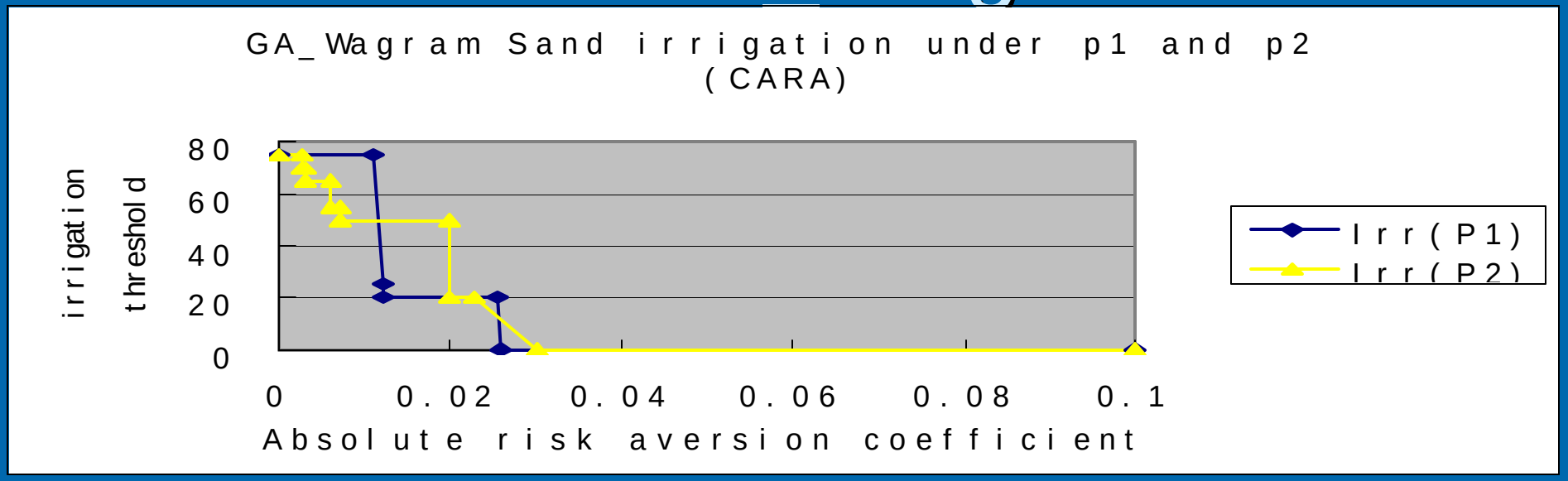
Data for Economic Model

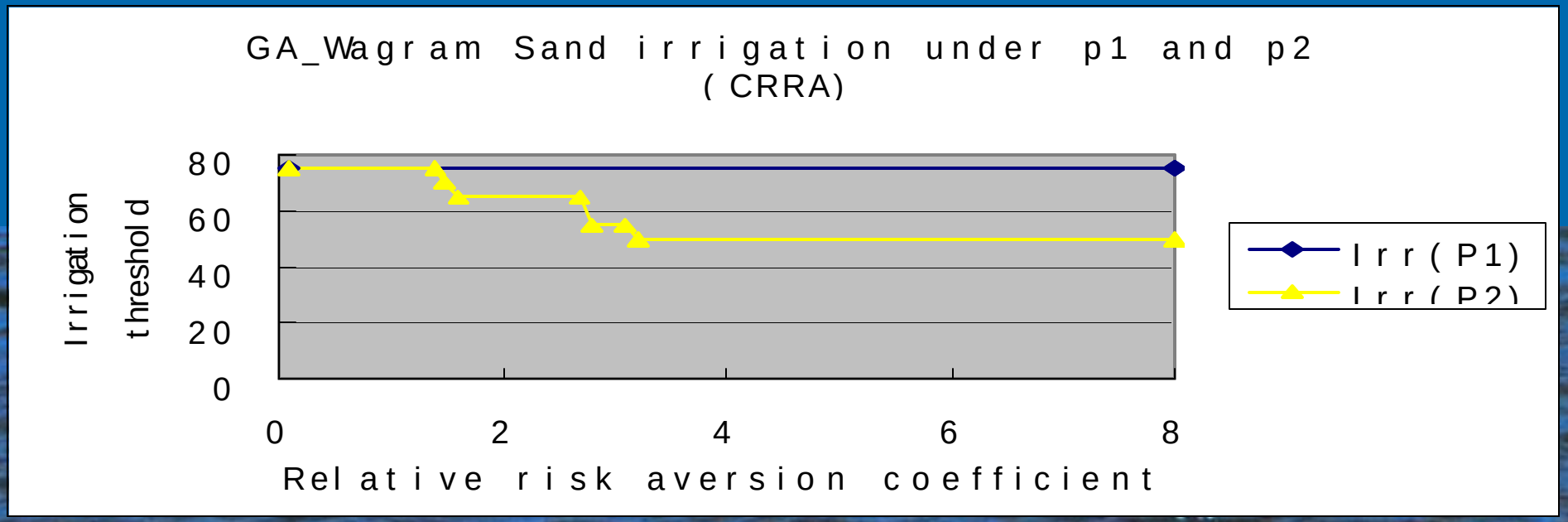
- Peanut price data
  - Two sets of peanut price: quota peanut price (P1) and additional (P2) peanut price.
- Irrigation cost
  - Pumping cost (estimated by fuel cost per acre foot of water)
  - Application cost ($/#)

Expected Result

- Irrigation is preferred to rainfed
- As risk aversion coefficient increases, water use decreases (Irrigation increases mean and variance of profit; higher risk averse producer puts more weight on variance)
- CRRA (Decreasing Absolute Risk Aversion) model may express a more aggressive water use behavior than CARA

Results for GA_Wagram Sand

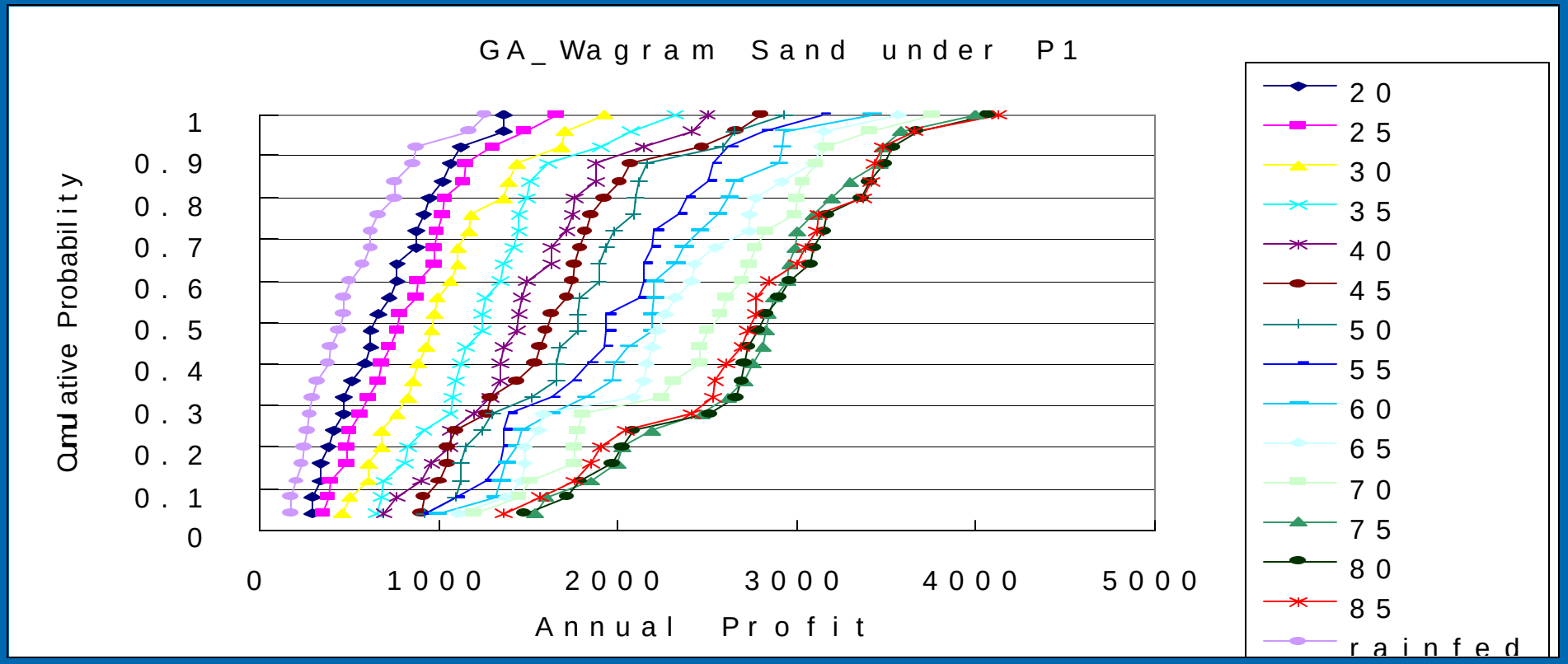


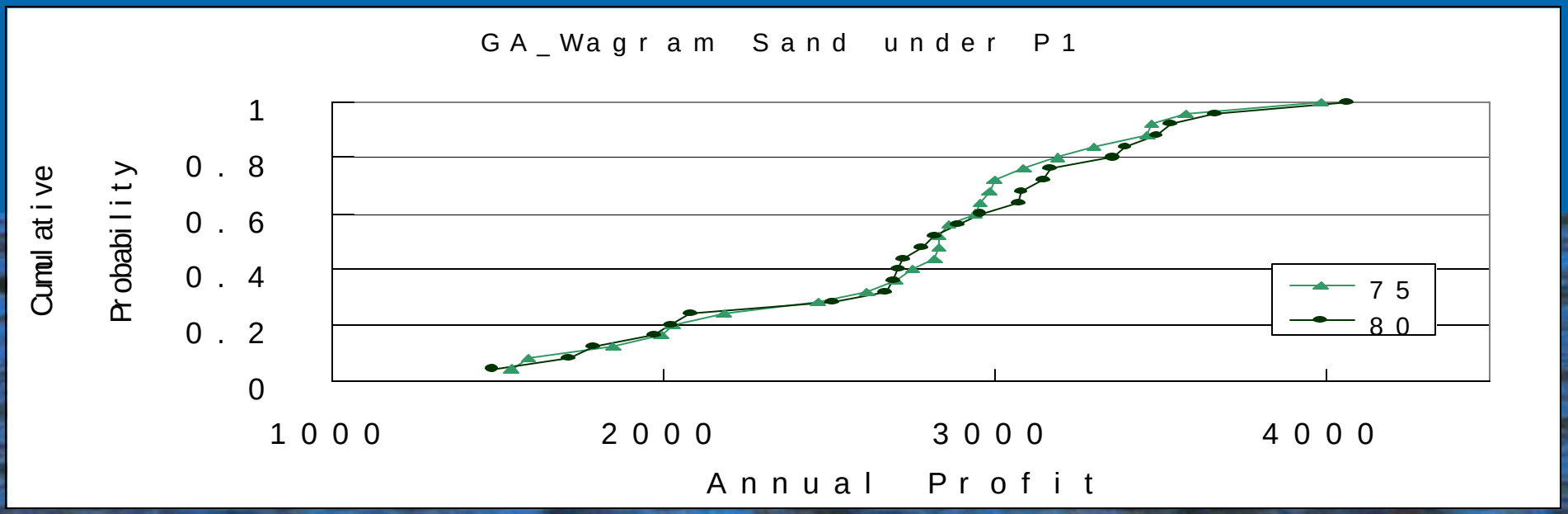


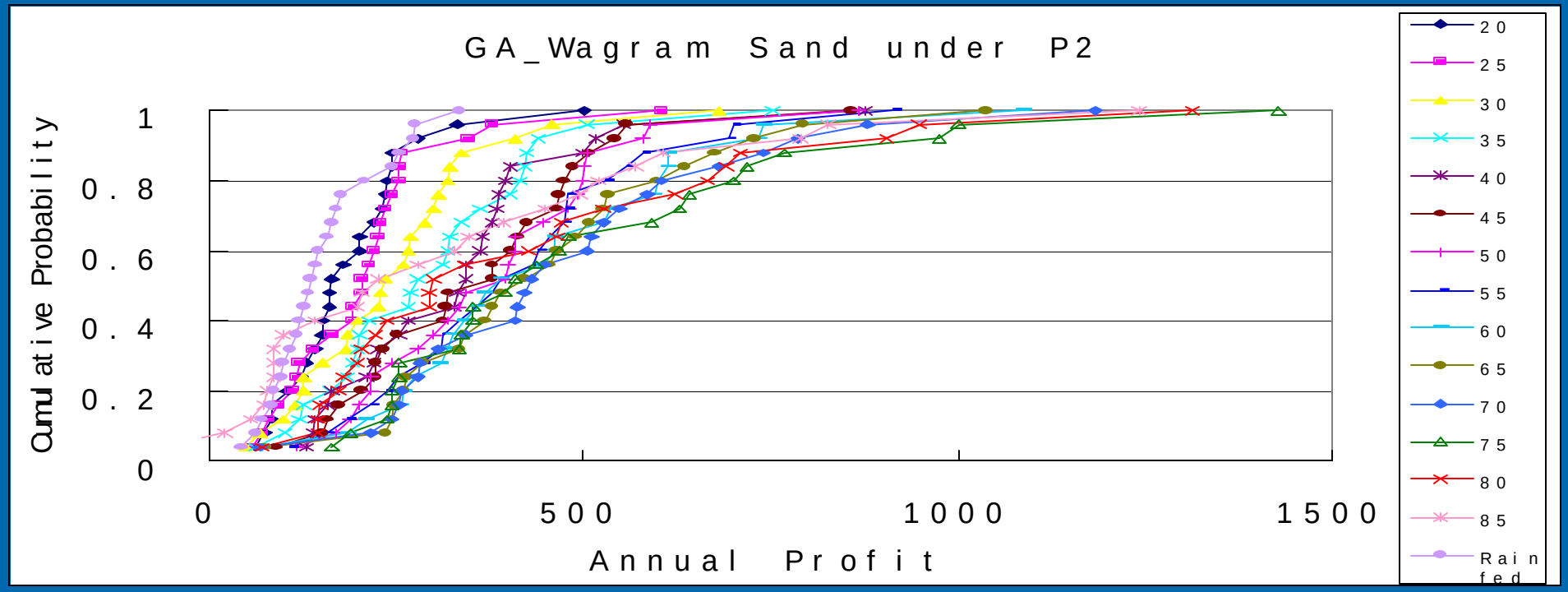
Discussions for CARA and CRRA

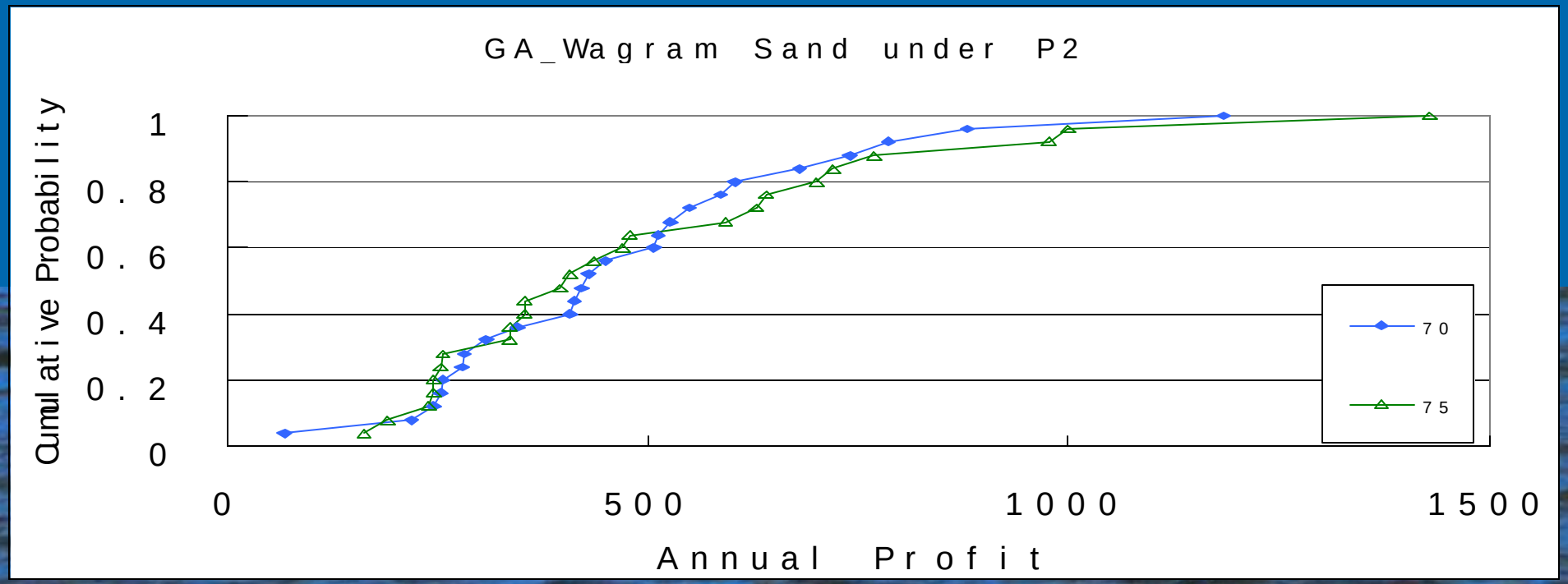
- Additional price (lower price, P2) , as r and ? increases
  - water use decreases under both models
  - small ?'s in r → large ?'s in water use under CARA (75%→65%→55%→50%→20%→0%)
  - water use exhibits a smoother progression under CRRA (75%→65%→55%→50%)

- Quota peanut price (higher price, P1), as r and ? increases,
  - Choose Rainfed over a wide range of r (irrational, FSD dominated) under CARA (75%→20%→0%)
  - water use constant (always the one with the highest profit, equal to risk neutral) under CRRA (75%)

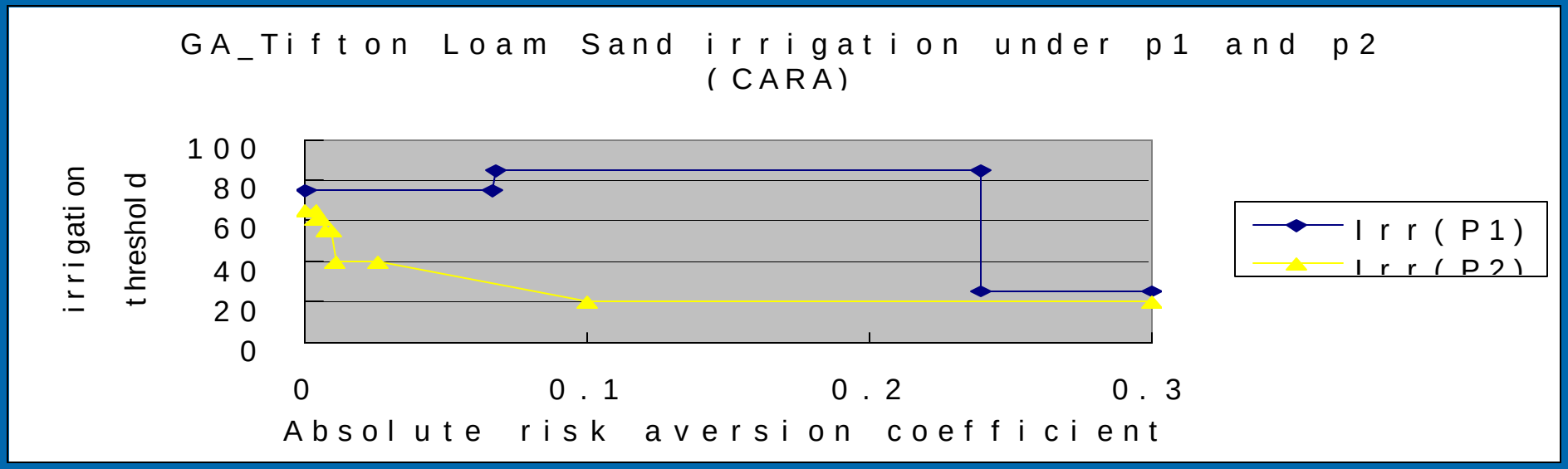

GA_Wagram Sand under P1
Cumulative Probability
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
1000
2000
3000
4000
5000
Annual Profit
20
25
30
35
40
45
50
55
60
65
70
75
80
85
rainfed

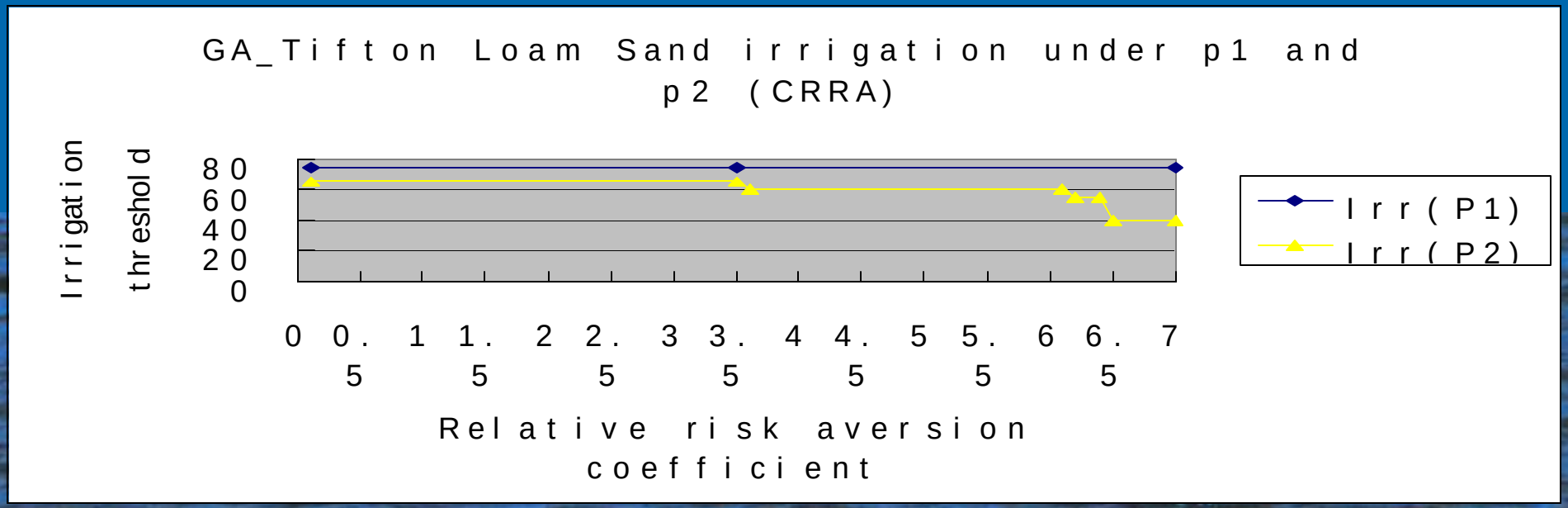

GA_Wagram Sand under P1
Cumulative Probability
1
0.8
0.6
0.4
0.2
0
1000
2000
3000
4000
Annual Profit
75
80

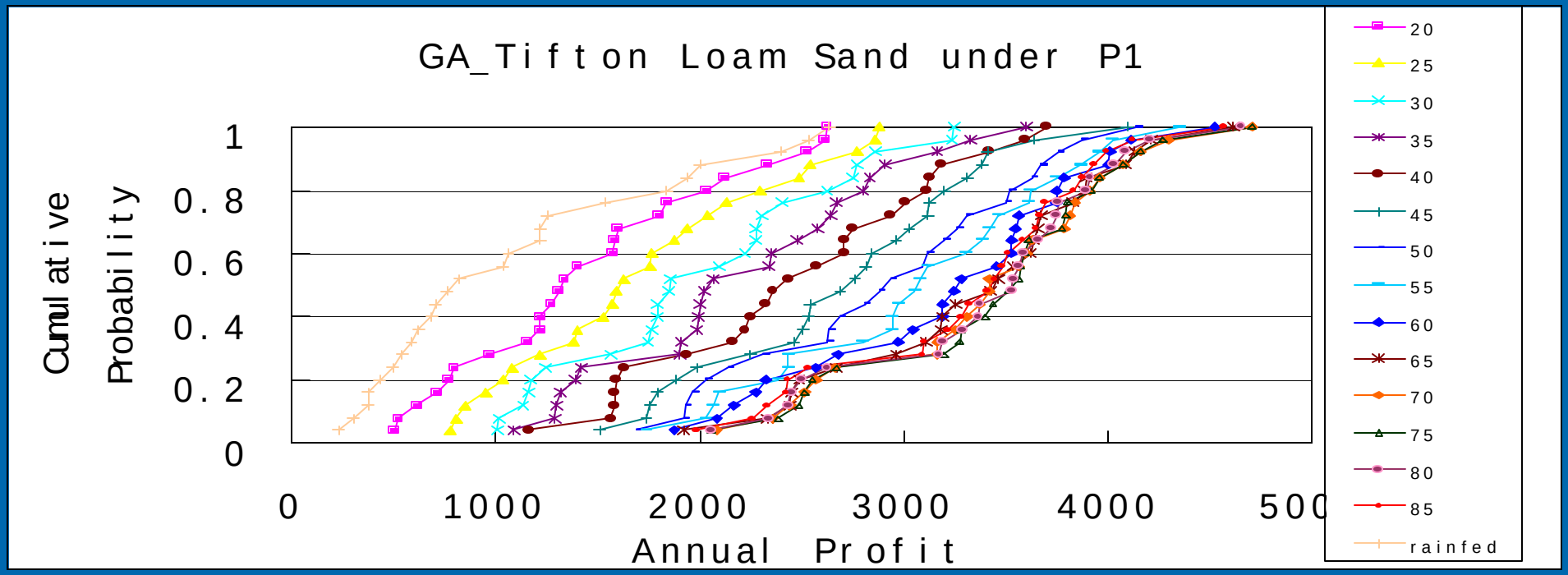

GA_Wagram Sand under P2
Cumulative Probability
1
0.8
0.6
0.4
0.2
0
0
500
1000
1500
Annual Profit
20
25
30
35
40
45
50
55
60
65
70
75
80
85
Rain fed

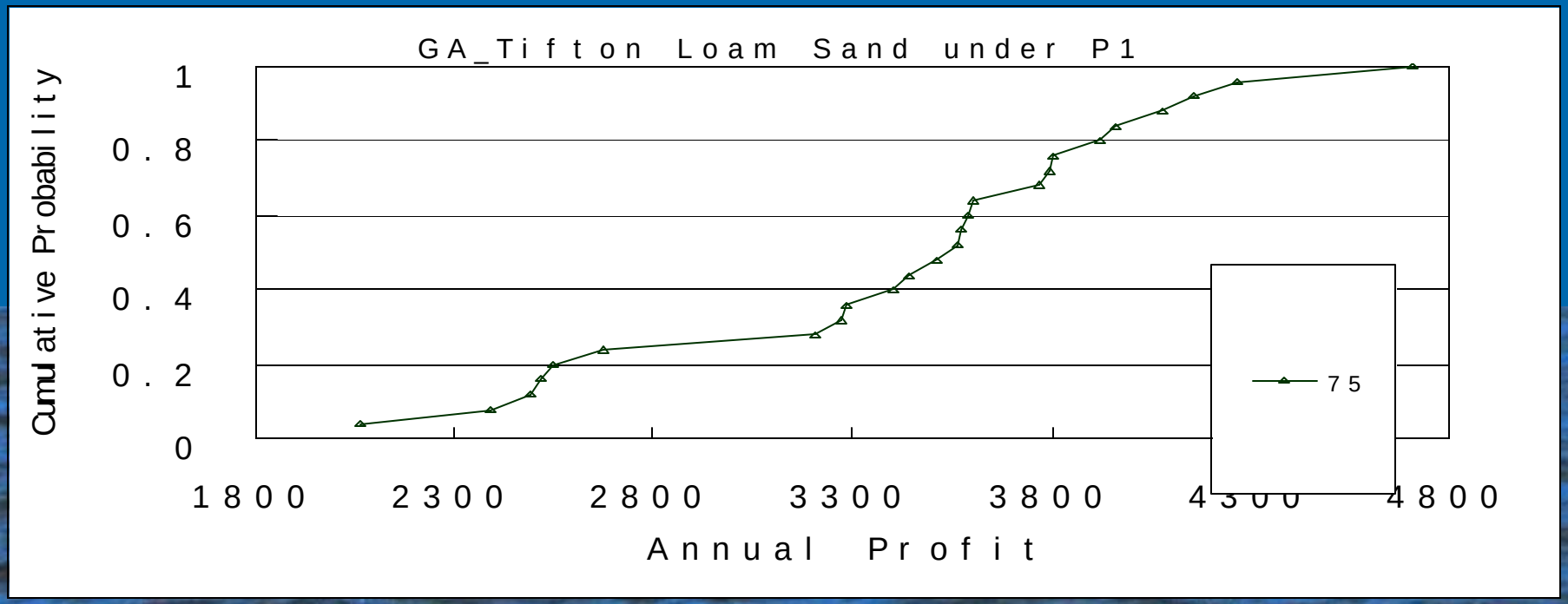

GA_Wagram Sand under P2
Cumulative Probability
1
0.8
0.6
0.4
0.2
0
0
500
1000
1500
Annual Profit
70
75

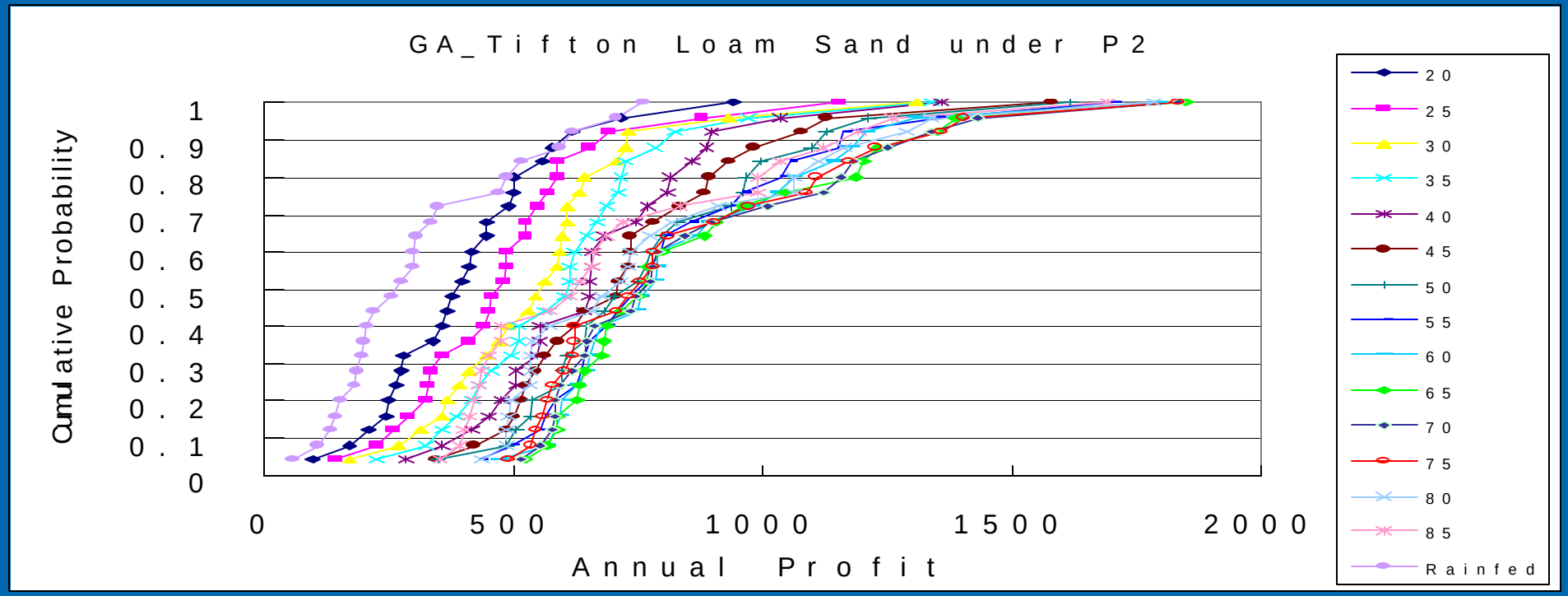
Results for GA_Tifton Loam Sand

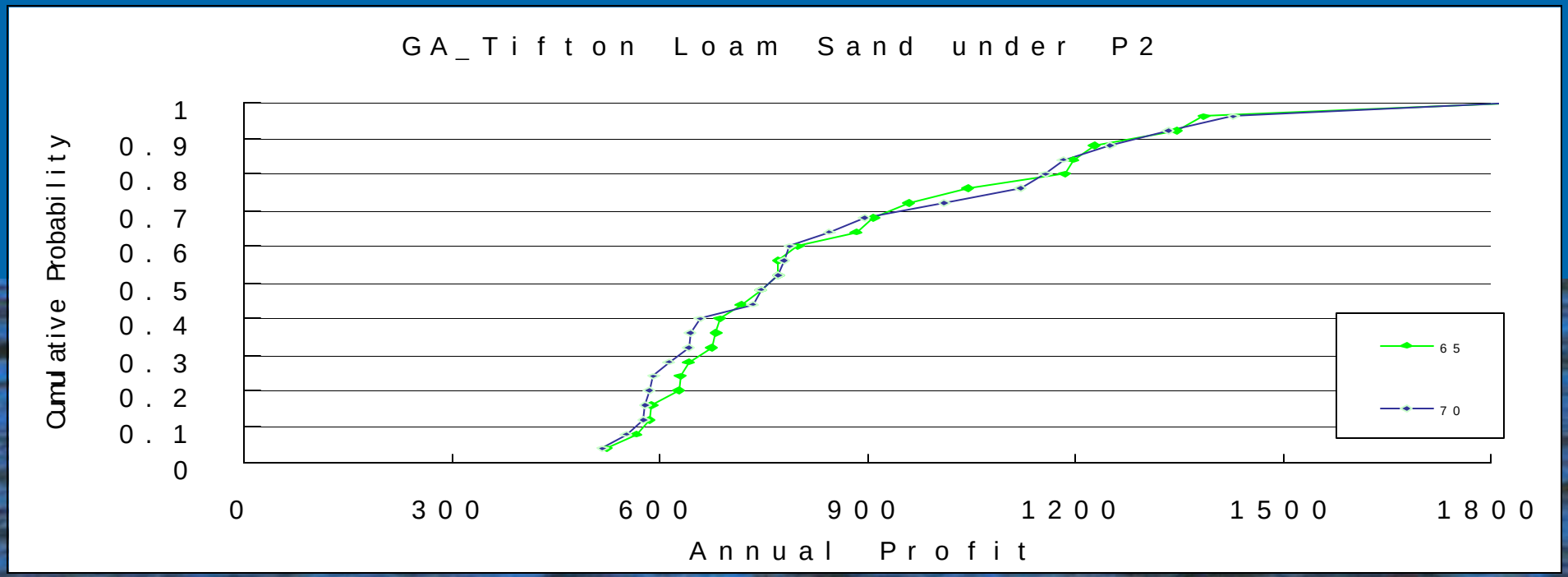




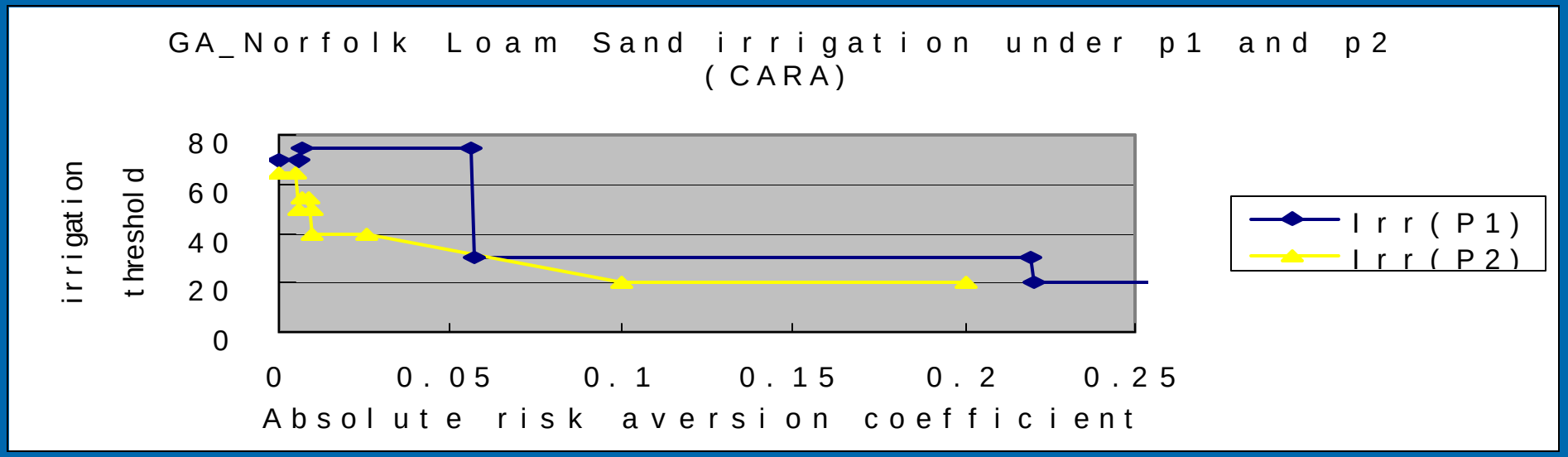

GA_Tifton Loam Sand under P1
Cumulative Probability
1
0.8
0.6
0.4
0.2
0
0
1000
2000
3000
4000
500
Annual Profit
20
25
30
35
40
45
50
55
60
65
70
75
80
85
rainfed

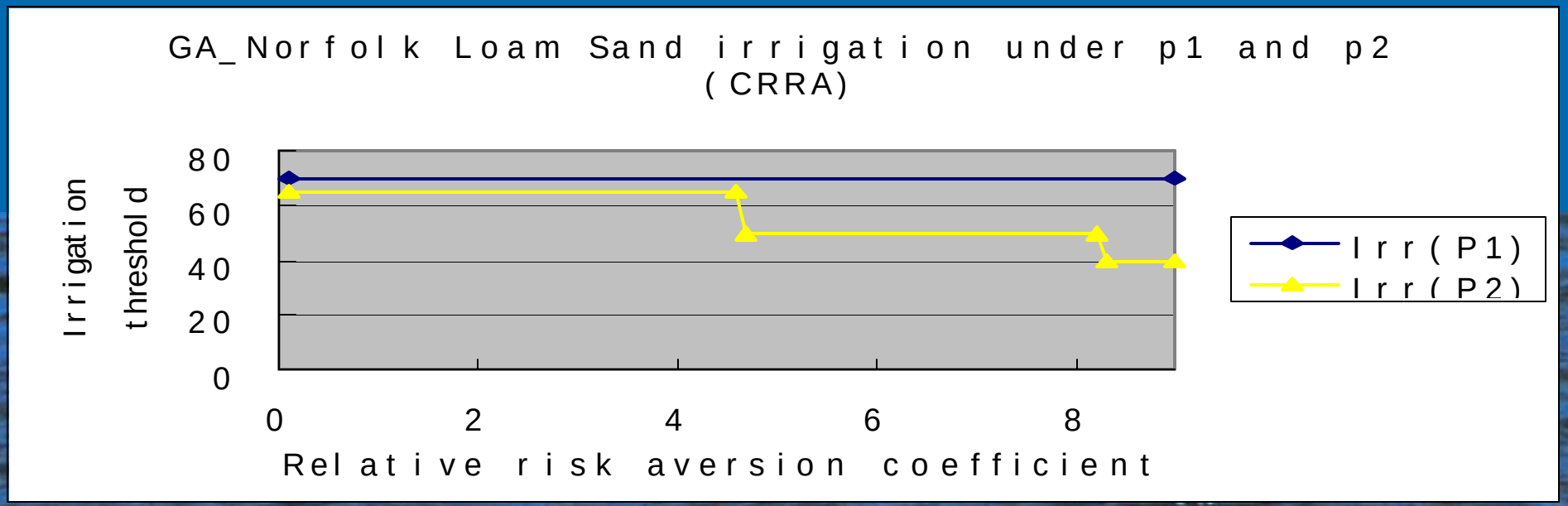

GA_Tifton Loam Sand under P1
Cumulative Probability
1
0.8
0.6
0.4
0.2
0
1800
2300
2800
3300
3800
4300
4800
Annual Profit
75

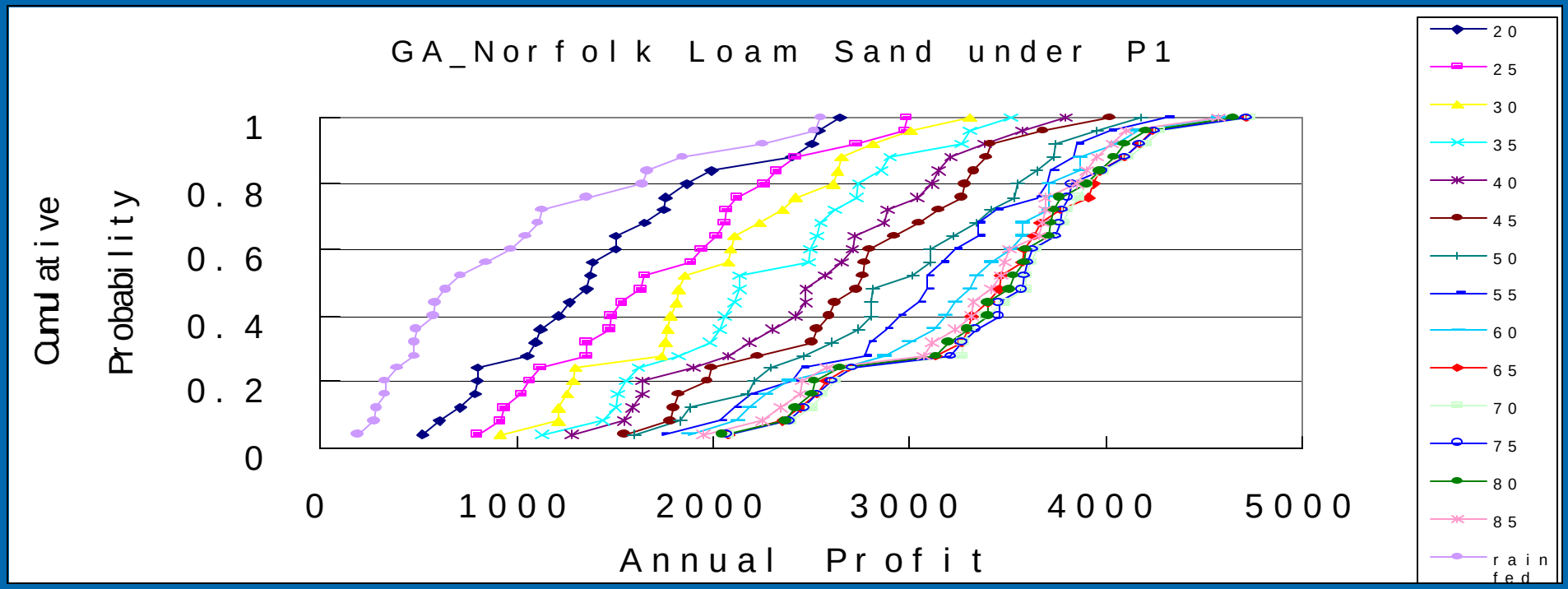

GA_Tifton Loam Sand under P2
Cumulative Probability
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
500
1000
1500
2000
Annual Profit
20
25
30
35
40
45
50
55
60
65
70
75
80
85
Rainfed

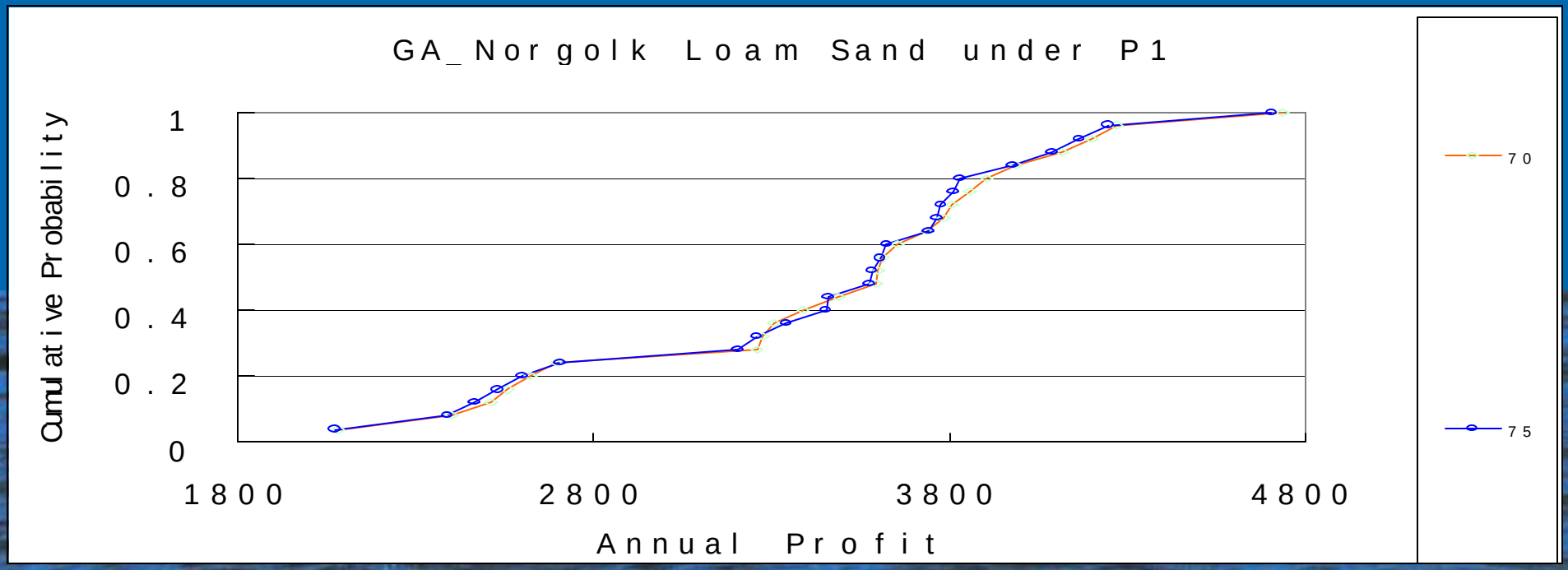

GA_Tifton Loam Sand under P2
Cumulative Probability
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
300
600
900
1200
1500
1800
Annual Profit
65
70

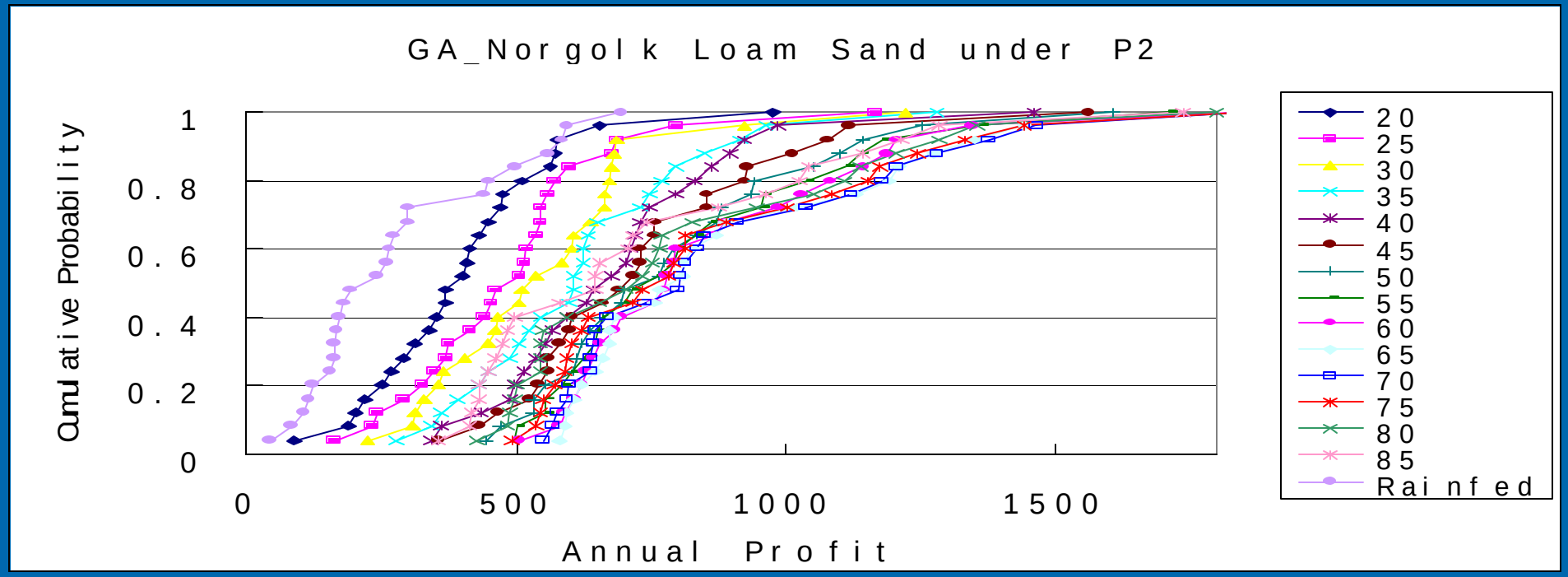
Results for GA_Norgolk Loam Sand

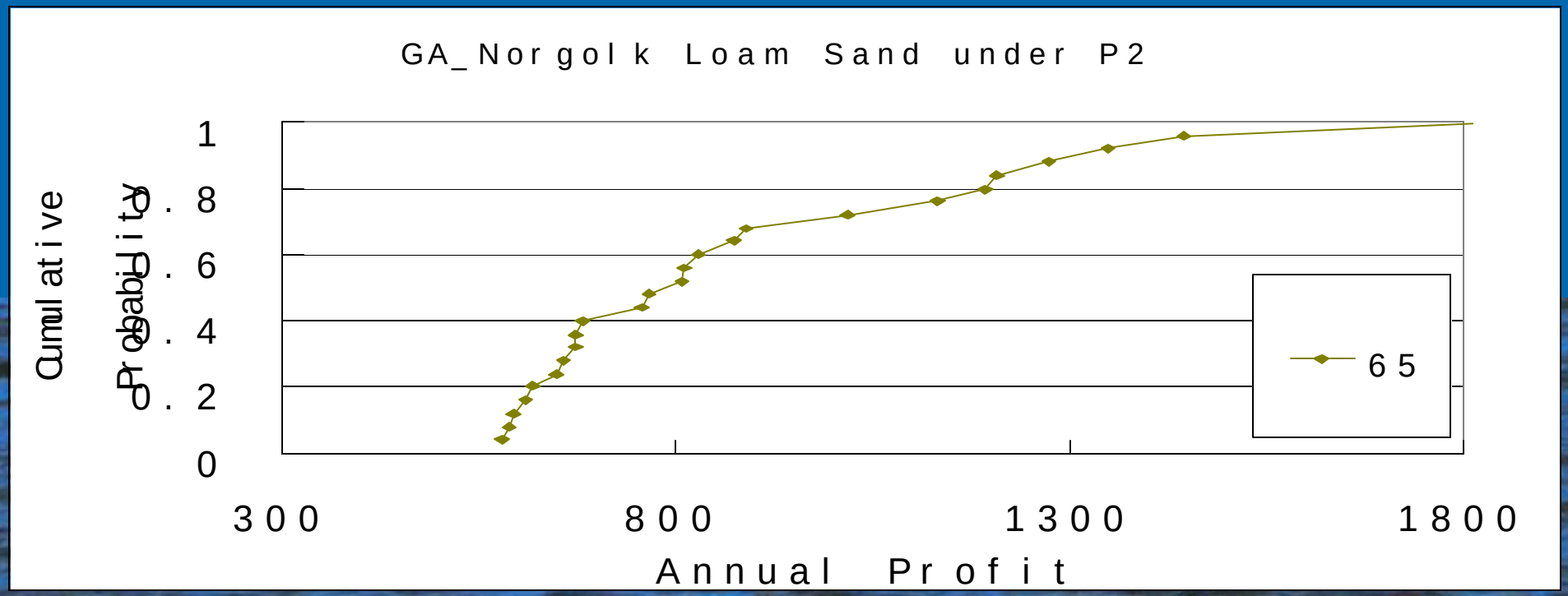




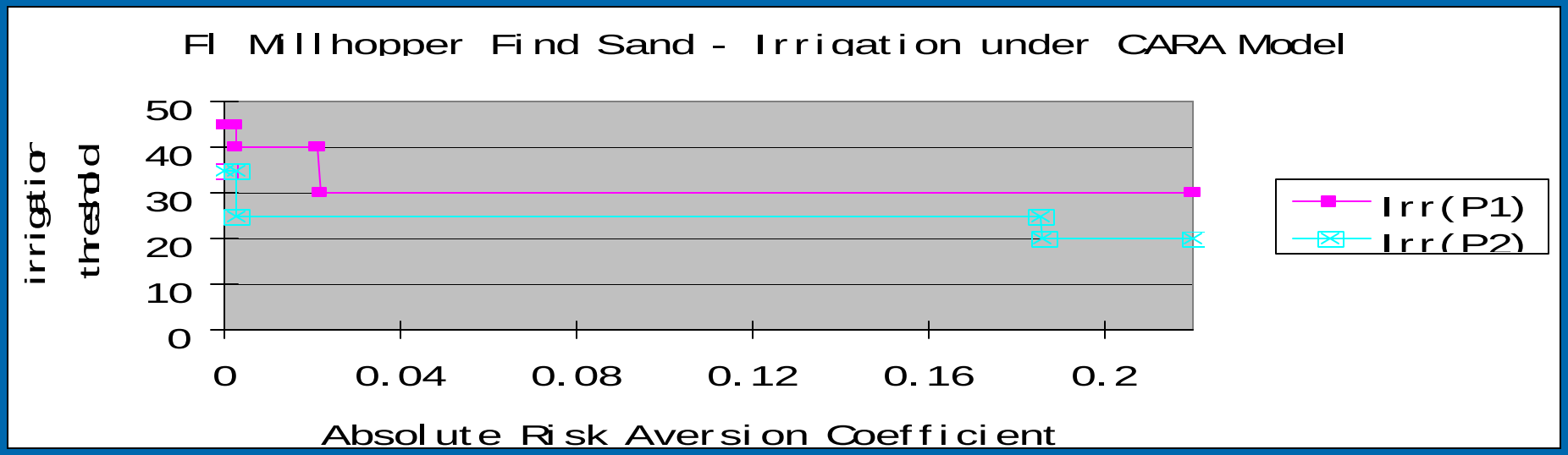

GA_Norfolk Loam Sand under P1
Cumulative Probability
1
0.8
0.6
0.4
0.2
0
0
1000
2000
3000
4000
5000
Annual Profit
20
25
30
35
40
45
50
55
60
65
70
75
80
85
rain fed

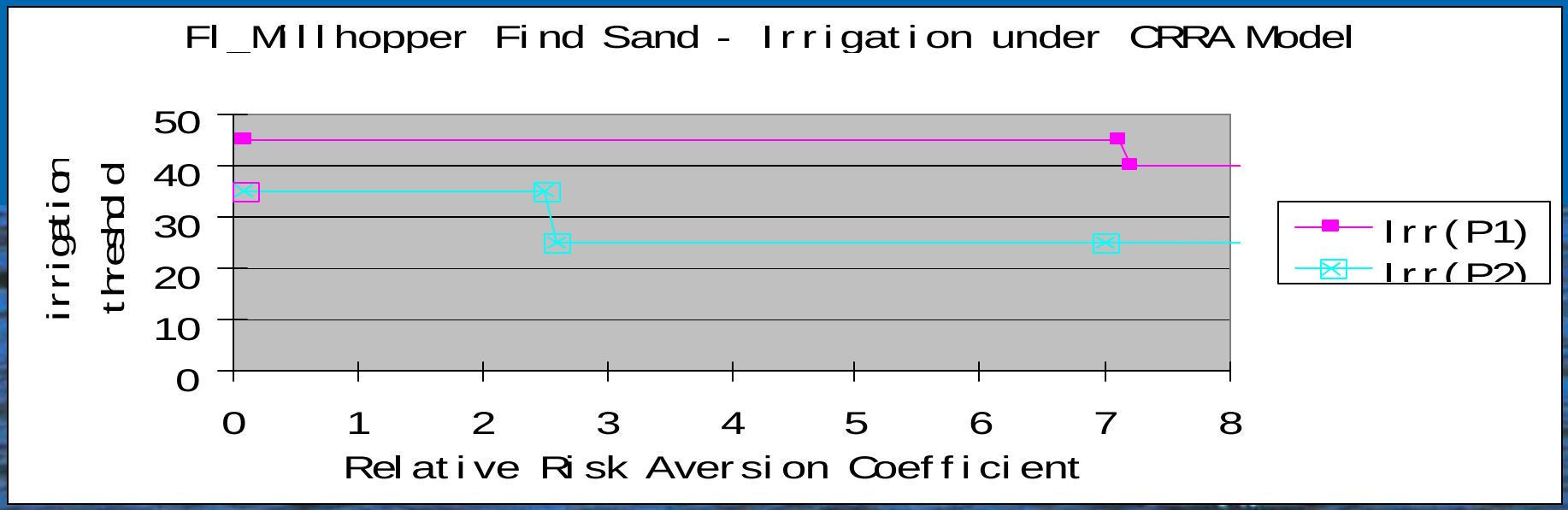

GA_Norgolk Loam Sand under P1
Cumulative Probability
1
0.8
0.6
0.4
0.2
0
1800
2800
3800
4800
Annual Profit
70
75

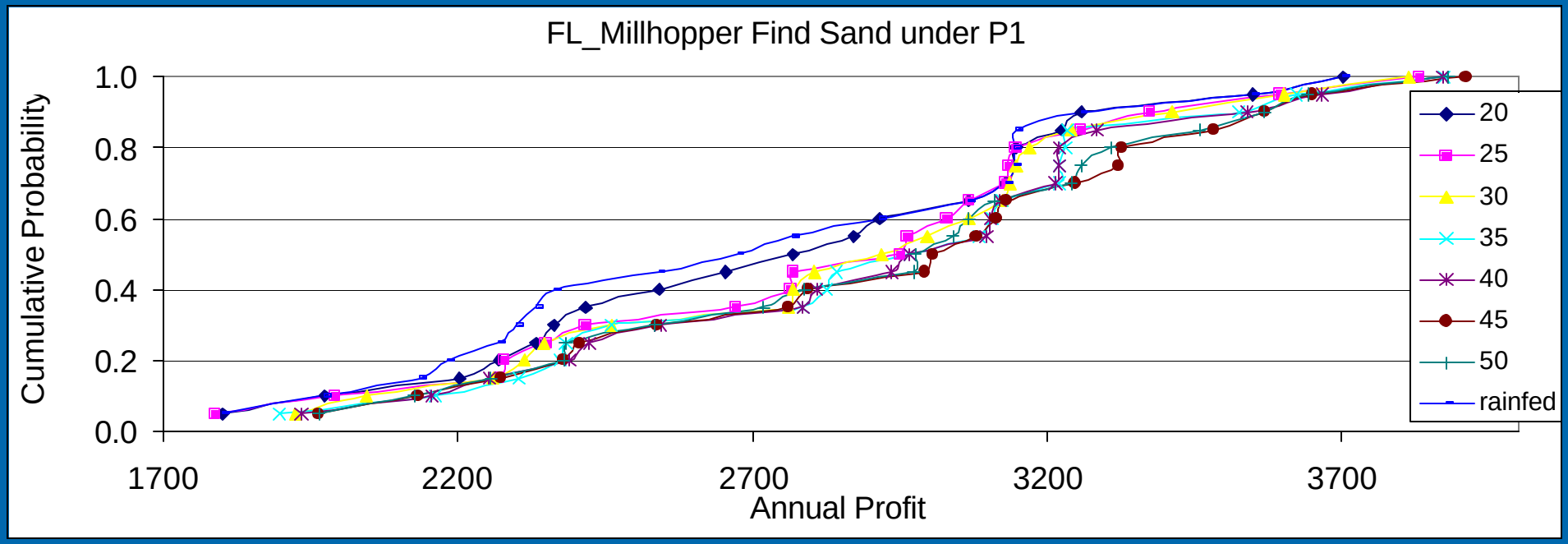

GA_Norgolk Loam Sand under P2
Cumulative Probability
1
0.8
0.6
0.4
0.2
0
0
500
1000
1500
Annual Profit
20
25
30
35
40
45
50
55
60
65
70
75
80
85
Rainfed

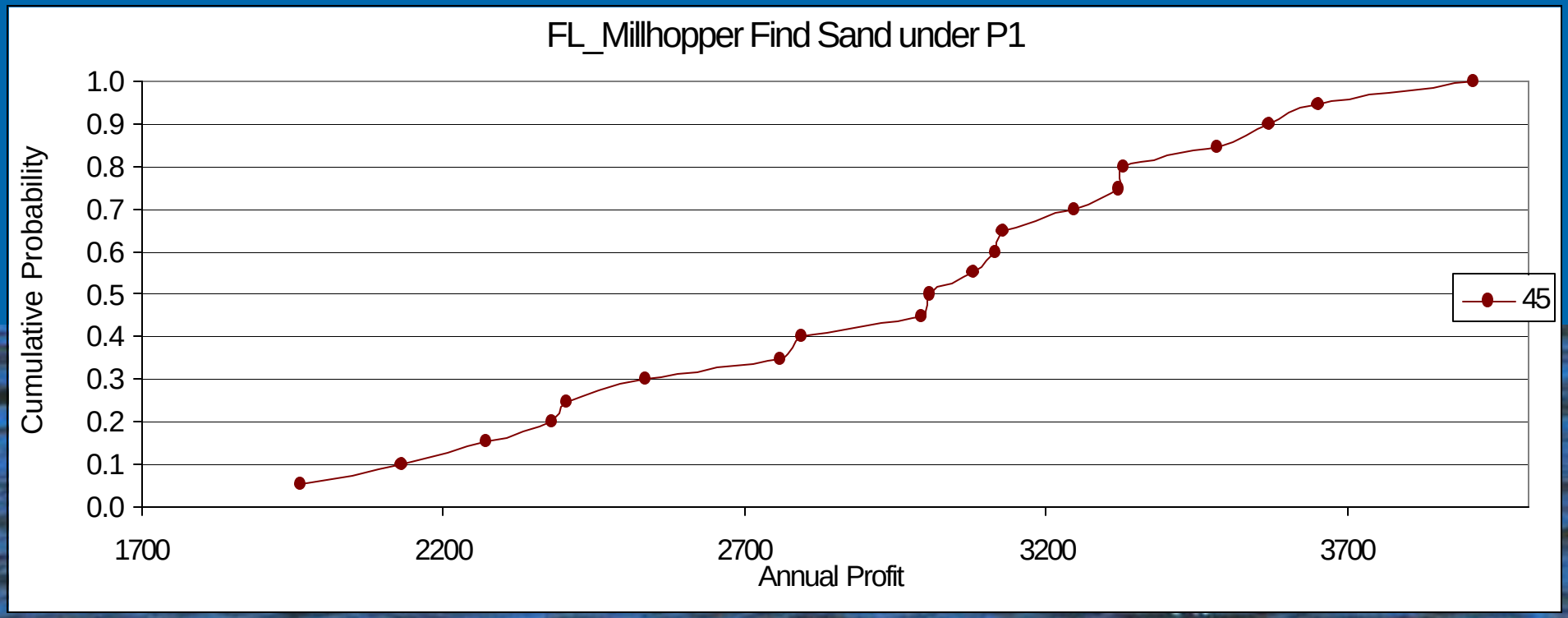

GA_Norgolk Loam Sand under P2
Cumulative Probability
1
0.8
0.6
0.4
0.2
0
300
800
1300
1800
Annual Profit
65

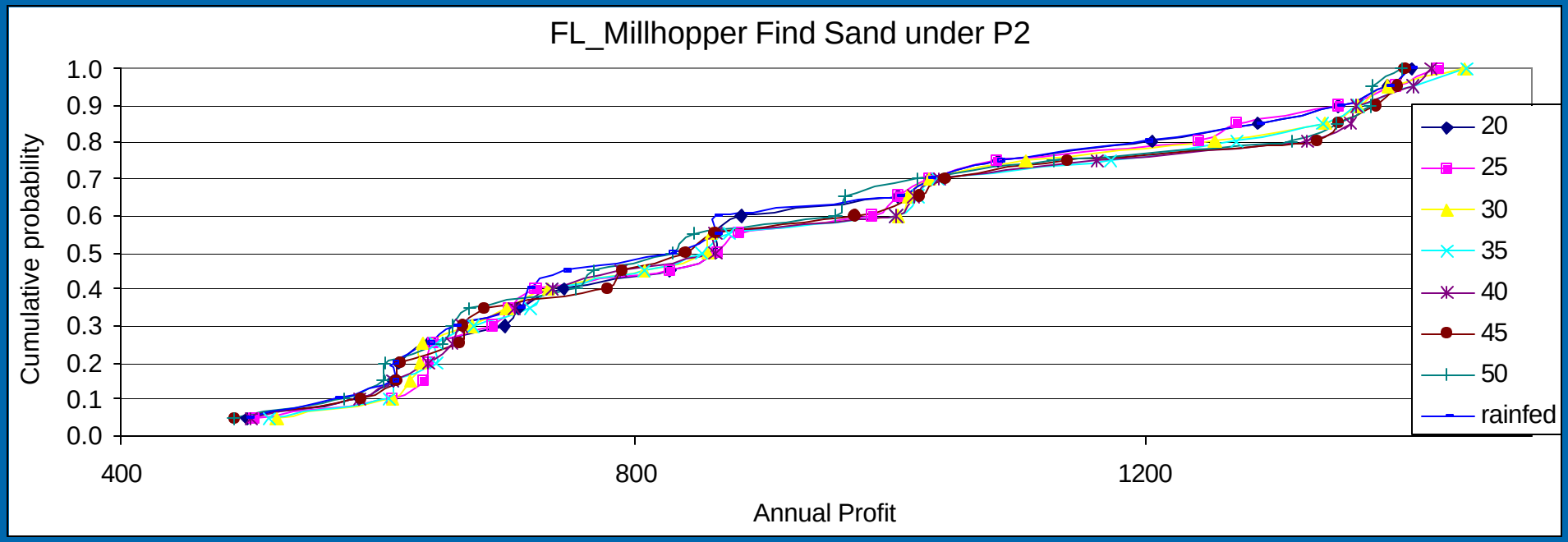
Results for FL_Millhopper Find Sand

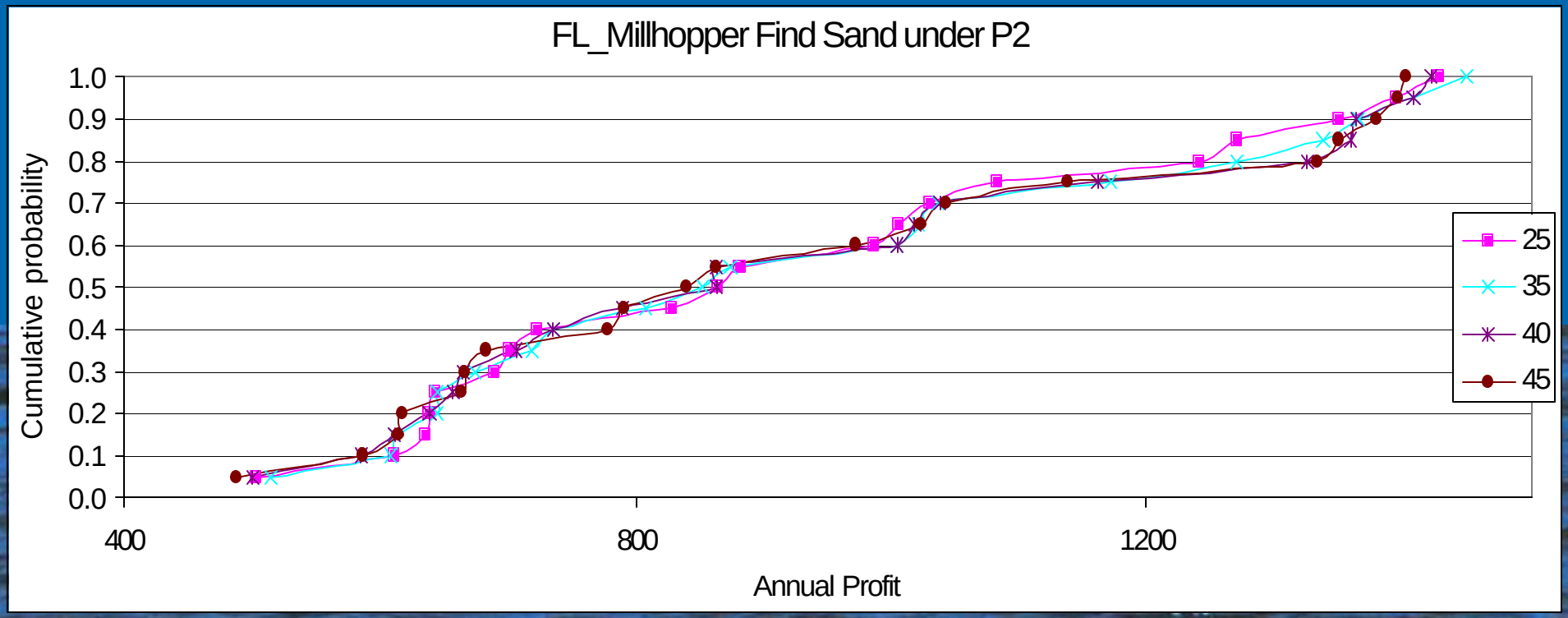




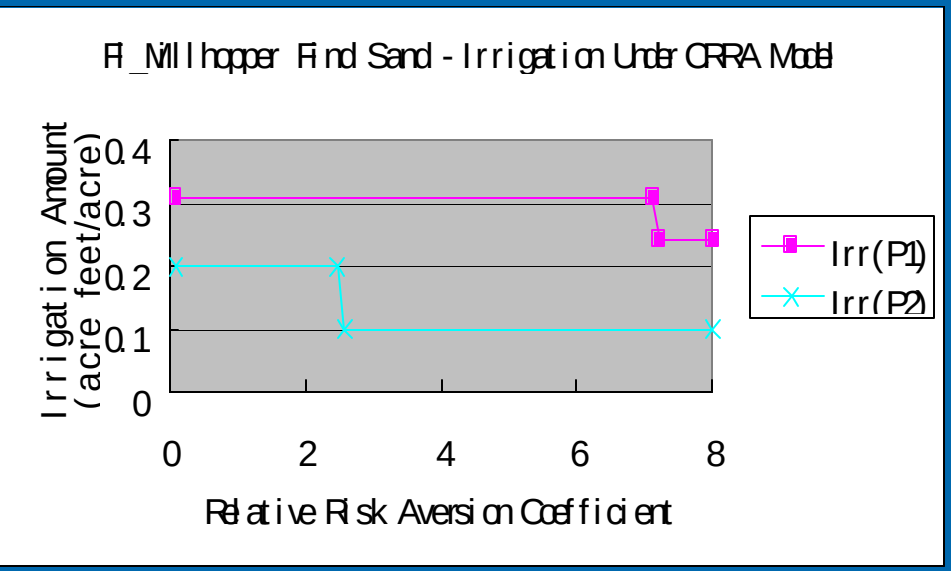

FL_Millhopper Find Sand under P1
Cumulative Probability
1.0
0.8
0.6
0.4
0.2
0.0
1700
2200
2700
3200
3700
Annual Profit
20
25
30
35
40
45
50
rainfed

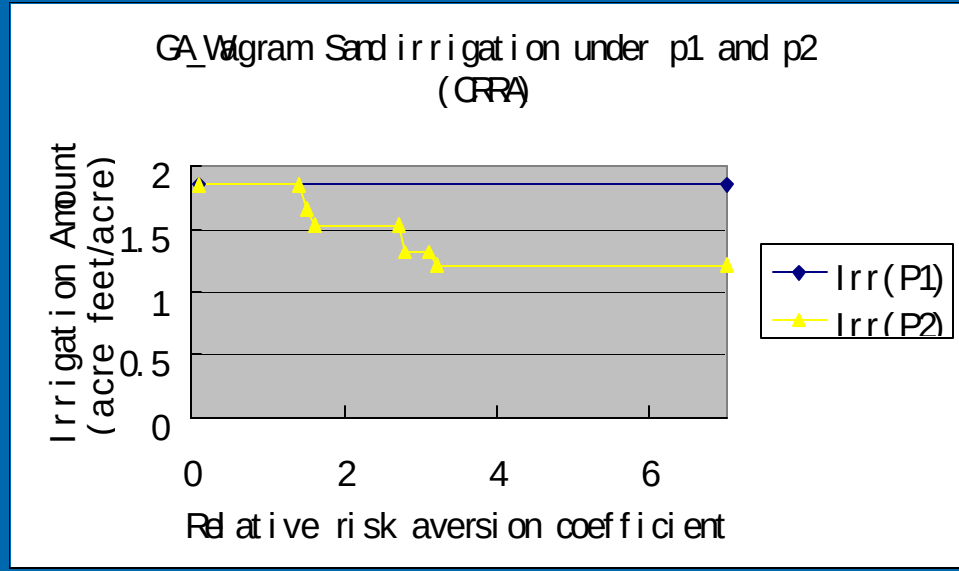

FL_Millhopper Find Sand under P1
Cumulative Probability
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
1700
2200
2700
3200
3700
Annual Profit
45

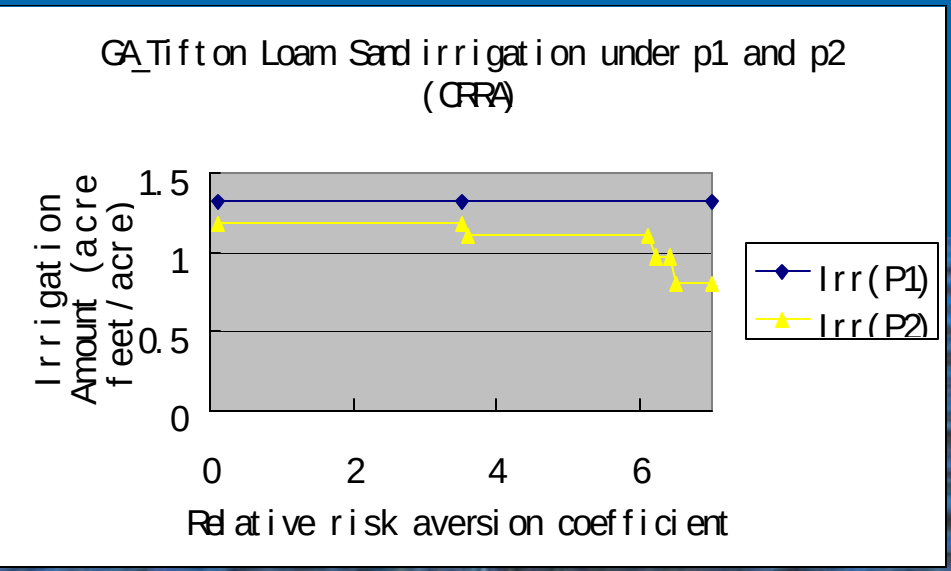

FL_Millhopper Find Sand under P2
Cumulative probability
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
400
800
1200
Annual Profit
20
25
30
35
40
45
50
rainfed

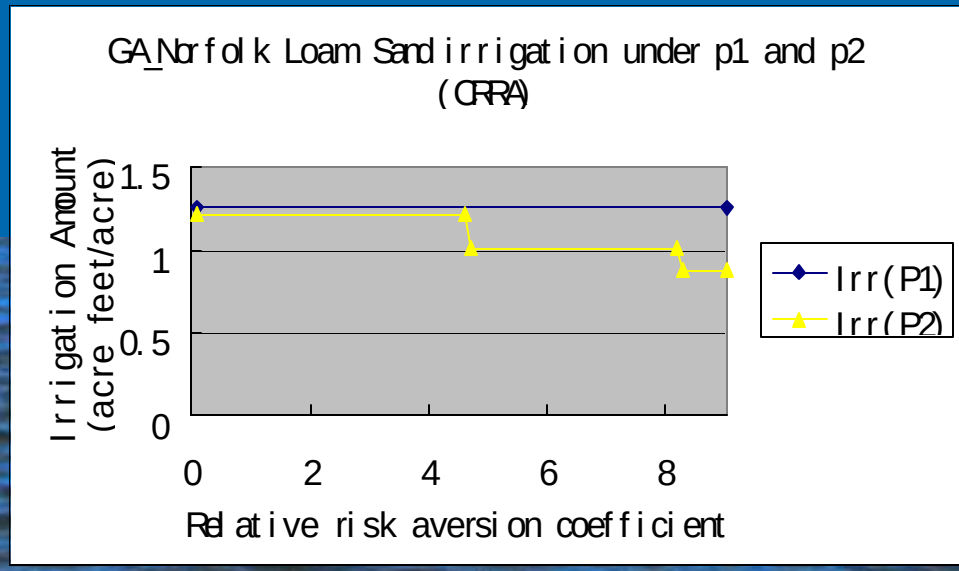

FL_Millhopper Find Sand under P2
Cumulative probability
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
400
800
1200
Annual Profit
25
35
40
45

Water Use Amount under CRRA

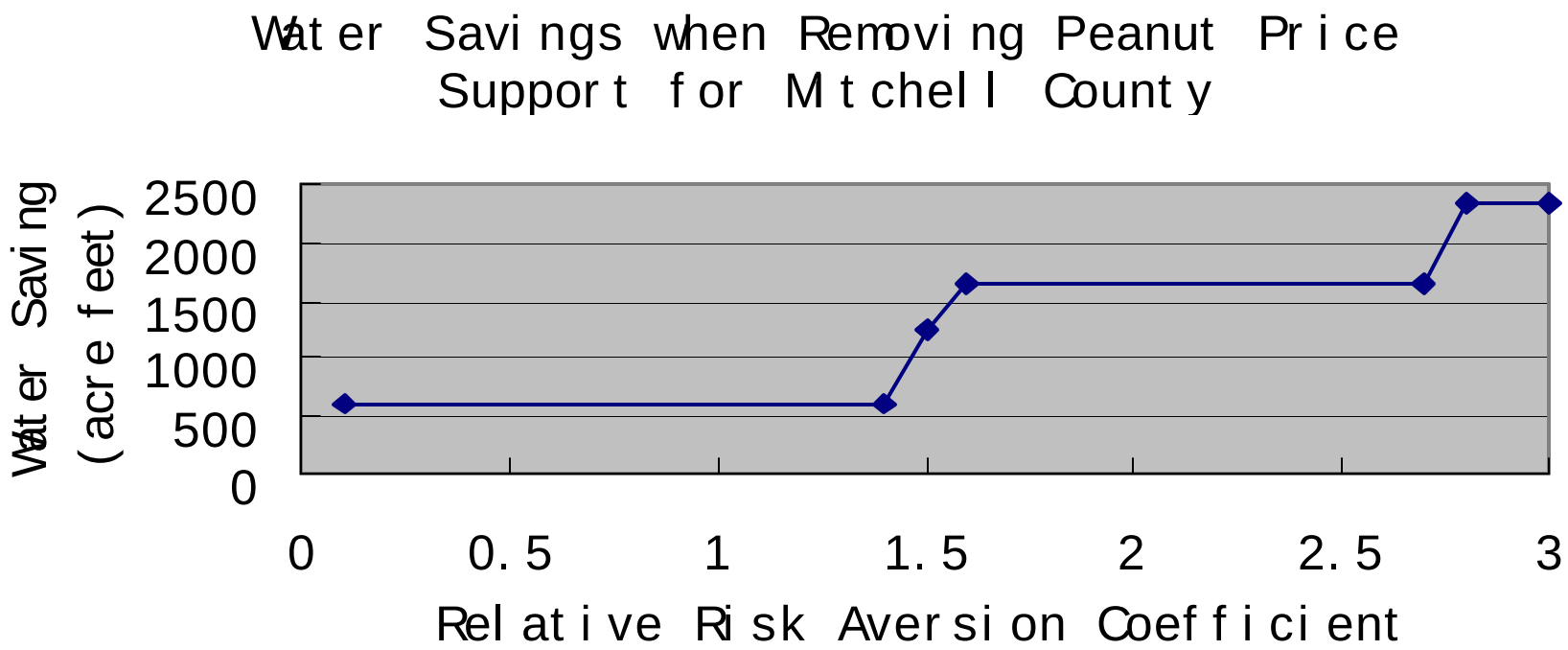








Impact of Recent Peanut Pricing Policy on Water Use



Conclusions

- We present two mean-variance utility models, which incorporate Decision Maker's risk aversion levels.

- Under both CARA and CRRA, as risk aversion level increases

  - water use decreases under lower price (P2)

  - water use under P2 is more sensitive to risk aversion level than under P1

Conclusions (Cont.)

- Transition across optimal irrigation thresholds is smoother under CRRA
  - results are not as sensitive to level of risk aversion
- CRRA provides more reasonable results than CARA, and is more consistent with Stochastic Dominance Analysis within a wide range of risk aversion.
- CRRA model suggests that recent changes in peanut pricing policy ➔ larger impact on water used by higher ? irrigators.

Direction of Future Work

- Change weather distribution
- Water pricing policy
- Expand to different crops in different states
- Financial instruments

- Thank you
- Questions?

Mean-variance (E-V) to express Expected Utility

- Under one of these two conditions:

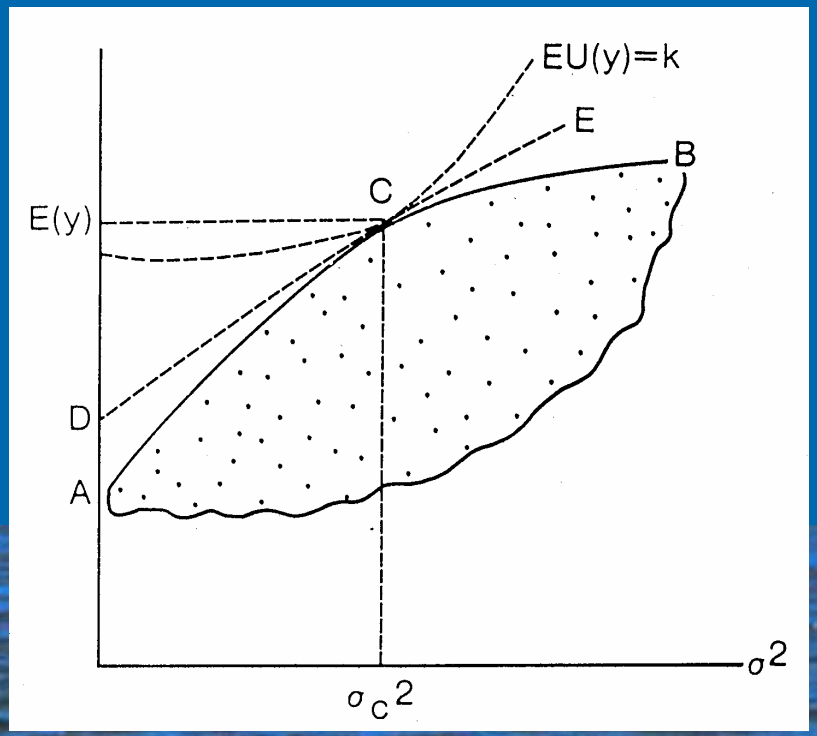
  1) Quadratic utility functions or

  2) All random variables are jointly elliptically distributed

  ← Location-Scale

(all attainable distributions differ only by location and scale parameters)

Mean-Variance (EV) indifference curve and feasible set



Mean-variance (E-V) to express Expected Utility

- Marginal Rate of Substitution betweens and μ, corresponds to the Arrow-Pratt measure of Absolute Risk Aversion (ARA):

. $$S(m,s) = -\frac{V_s(m,s)}{V_m(m,s)} \leftrightarrow R_a(y) = -\frac{u''}{u'}$$

- The partial derivative of S with respect to μ indicates the nature of ARA, specifically, if

$$S_m < 0 \text{ (DARA)}. \quad S_m > 0 \text{ (IARA)}, \quad S_m = 0 \text{ (CARA)}$$

- $$S_t = \frac{\partial S(tm, ts)}{\partial t}$$ indicates the nature of relative risk aversion or RRA,

i.e., $$S_t < 0 \text{ (DRRA)}. \quad S_t > 0 \text{ (IRRA)}, \quad S_t = 0 \text{ (CRRA)}$$