HOW GRANULAR MEDIA STRAIN PARTICLES SMALLER THAN PORE THROATS

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❖ Research Motivation

➢Modeling the retention of colloidal particles in soils is important to understanding water contamination from viruses, bacteria or contaminants adsorbed on colloids. Particles transported by fluid remain in the soil when they arrive at constrictions in the pore space too small to admit them. Theories of this phenomenon predict that dilute concentrations of particles smaller than the smallest nominal pore throats should migrate without retention. But many investigators have reported that particles too large to undergo filtration and too small to be strained are nevertheless retained in a porous medium.

➢We propose a geometric explanation for this observation, namely that particles are strained not just in throats between three grains, but also in **gaps between pairs of grains**. We support this explanation by characterizing such gaps in model soils (dense random packings of spheres).

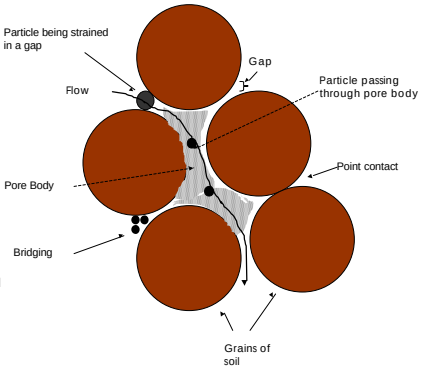

Figure 1: Sketch of pore space in a soil showing the retention of particles between grains. (Cut of a 3 dimensional figure; see Fig. 2).

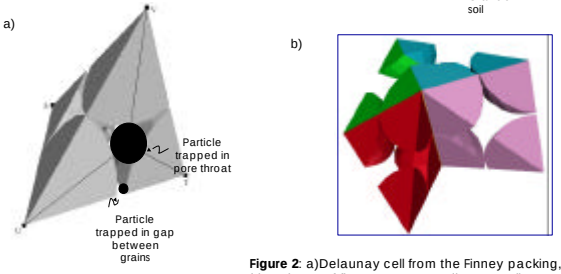

Figure 2: a)Delaunay cell from the Finney packing, and b) A cluster of five pores surrounding a small gap between grains

❖ Statistical characterization of the pore space in model soils

➢The real packing measured by Finney and different computed-generated sphere packings showed similar statistics regarding number of neighbors and gap densities. All the spheres in the packings have the same radius.

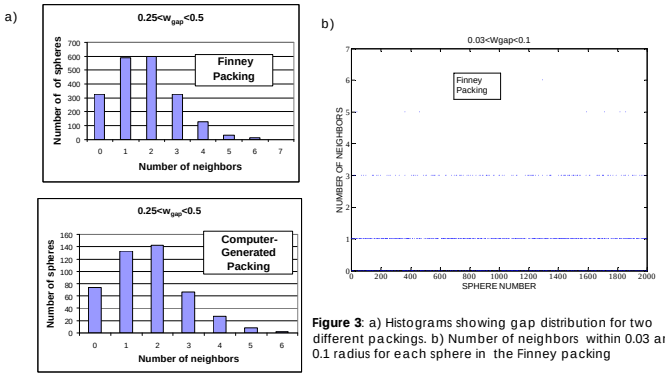

Figure 3: a) Histograms showing gap distribution for two different packings. b) Number of neighbors within 0.03 and 0.1 radius for each sphere in the Finney packing

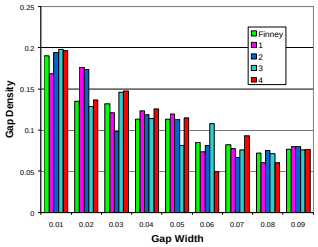

Figure 4: Gap density for different gap widths for the Finney packing and 4 random packings

➢Particles 3 to 9 times smaller than pore throats underwent appreciable retention in sand filters. Of **primary interest** are gaps having widths between 1/9 to 1/3 of the of the biggest pore throat diameter. These widths are **0.03 and 0.1*R***, *R* being the sphere radius. The number density of such gaps is about 0.2 per R^3 bulk volume in the model soils. This is enough to trap a significant number of particles in the size range of interest. The density of gaps is comparable to the density of small throats, which is about 0.3 per R^3. For comparison, the number density of point contacts is about 0.9 per R^3.

❖ Flow Rate Through Gaps

➢A particle of diameter $d=w_{gap}$ will be strained only if the stream line carrying it passes through this minimum constriction. The probability of this event is infinitesimal. In contrast a particle of $d> w_{gap}$ will be strained if it passes within some finite distance **a** of the minimum constriction. The probability of entering this range of capture is finite and is measured by the flow rate within this distance of the minimum constriction. Using the slit approximation the local flow rate relevant to particles of size d Is:

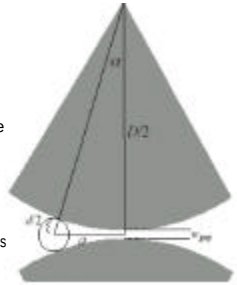

Figure 5: Scheme of the range of capture

$$q=2aw_{gap}\,u_{gap}$$

➢ Steady flow through the model soil creates a pressure variation from pore to pore. The volumetric flow rate through a pore throat (Fig. 2) is the product of its conductance and the difference in potential in the two pores connected by the throat. The steady-state flow field is determined from the requirement that mass does not accumulate in any pore. We use the resulting pore pressures and gap conductances to estimate flow velocity through each gap.

$$\sum_{j=1}^{4} q_{ij} = 0 \qquad \sum_{j=1}^{4} g_{ij}(P_i - P_j) = 0$$

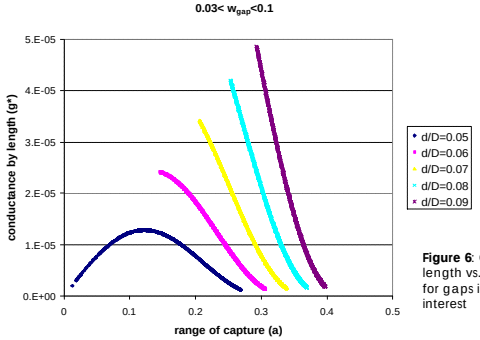

Figure 6: Conductance by length vs. range of capture for gaps in the range of interest

➢g* is proportional to the gap width cubed times the range of capture. For a given gap width the range of capture increases as the particles strained become bigger.

❖ Pressure Gradient in Gaps

➢The centers of five neighboring Delaunay cells containing a gap are in the same plane (Fig. 2b). The objective is to relate pressure with spatial coordinates to estimate the pressure gradient in this plane.

➢ After translation and rotation the plane coincides with the plane z=0.

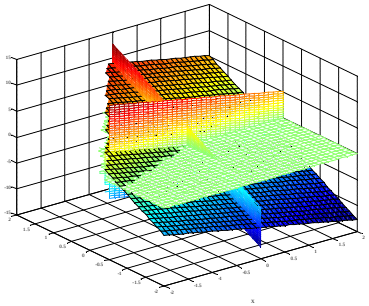

Figure 7: Plane containing the centers of 5 neighboring Delaunay cells before translation and rotation.

➢ Once we have rotated the plane and have the pressure as a function of only 2 coordinates **x** and **y** it is easy to plot pressure contours. The center of the gap is somewhere in the plane.

➢The pressure fit the spatial coordinates in the form **P = Ax +By + C**

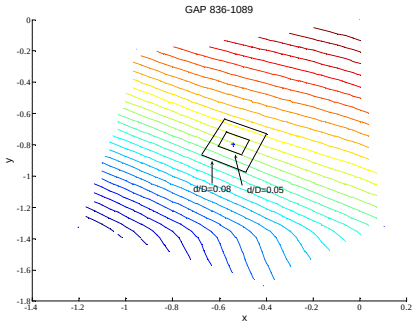

Figure 8: Pressure contours. The center of the gap and the radius of capture are shown. The range of capture gets bigger as the ratio d/D increases

➢**Future work:** From here we will calculate the local gradient in potential in the plane normal to the line that joins two grains separated by a gap. This potential drives flow through the gap. The flow through the gap is an important quantity in theories of straining.

$$u_{gap} = -\frac{w_{gap}^2}{12}\frac{\nabla P}{m} \qquad q = 2a \cdot w_{gap} \cdot u_{gap}$$

❖ Conclusions

➢These data enable a prediction of particle trapping in gaps as a function of particle size.

➢We obtain a conservative estimate for the volume of particles that can be trapped in gaps. The predictions are consistent with experiments, providing new insight into the mechanism of straining.

➢The frequency distribution of gaps between grains and the flow rates through each gap for a given particle size enter the continuum model via the local flow distribution. Data from pore throats will be combined with data from gaps to form the complete flow distribution